Lec 17: 03/21/2012

Binary Systems and Accretion from a Companion Stars

In many cases, isolated compact objects become visible because of accreting plasma from the surrounding medium. Super massive black hole at the center of galaxies are examples of such a case. There are however common situations where high-energy sources grow by absorbing matter from a companion in tight binary systems. Examples of such cases include nova and X-ray bursts.

Lets start our discussion of accretion in binary systems by considering the motion of a test particle in such systems. Consider two objects of mass M_1 and M_2 orbiting around each other. According to Kepler's law:

$$4\pi^2 a^3 = G(M_1 + M_2)\, T_{orb}^2$$

Here T_{orb} is the period of orbital motion and a is the semi-major axis of the orbit. Going to the frame that corotates with the object, the forces acting on the test particle are:

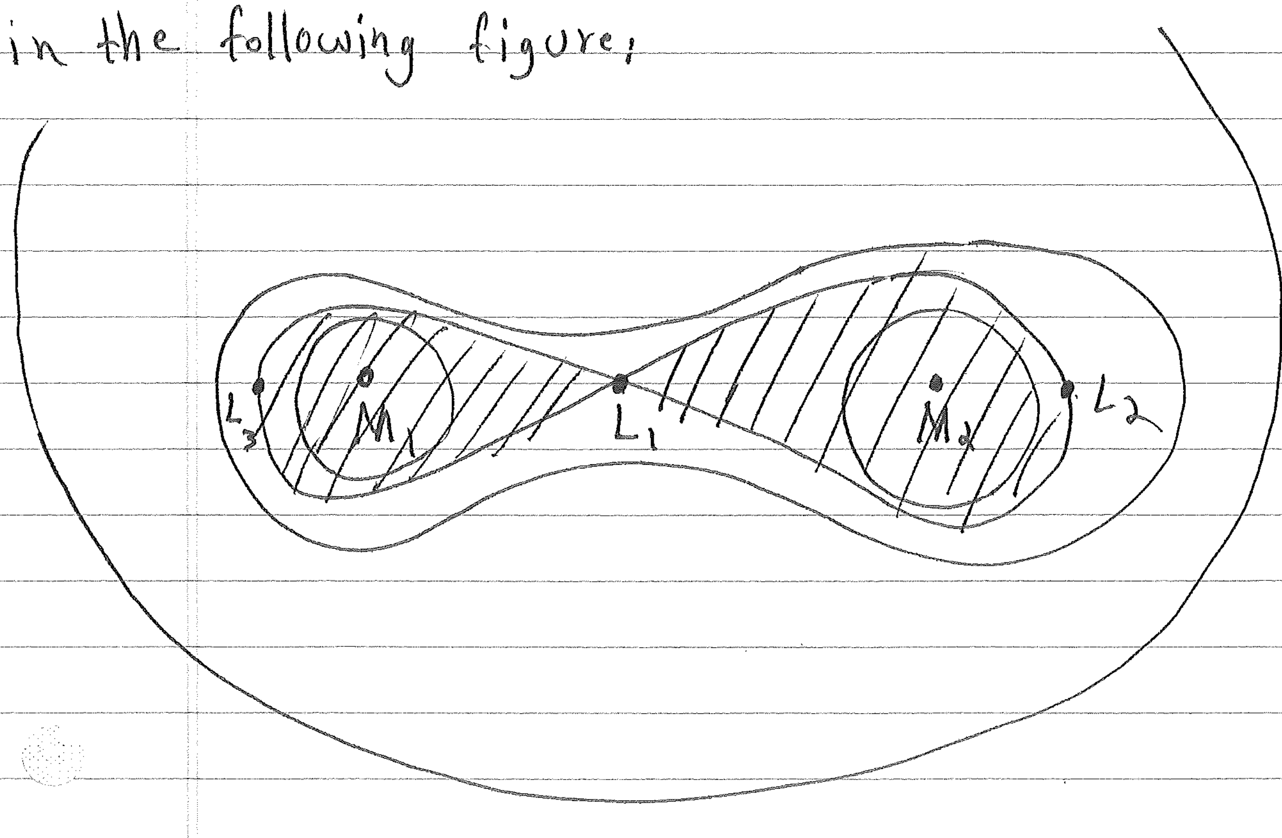
$$\vec{F}_1 = \frac{-GM_1 m}{|\vec{r}-\vec{r}_1|^3}(\vec{r}-\vec{r}_1) \quad , \quad \vec{F}_2 = -\frac{GM_2 m}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2)$$

$$\vec{F}_{cf} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

Here the position vectors are relative to the center-of-mass of the binary, and $\vec{\omega}$ is the angular velocity vector. Note that $\vec{F}_{cf}$ is a fictious force. The total potential of the test particle, called Roche potential, can then be written as:

$$\Phi_R(\vec{r}) = \frac{-GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}|\vec{\omega}\times\vec{r}|^2$$

One can find useful information about the behavior of the test particle by drawing equipotential surfaces. For $r \gg a$, equipotential surfaces are circles whose center are at the center-of-mass of the system. When $|\vec{r}-\vec{r}_1|$ is very small, equipotential surfaces are circles whose center are at the position of M_1. Similarly, when $|\vec{r}-\vec{r}_2|$ is very small, equipotential surfaces are circles whose center are at the position of M_2. The situation can be summarized in the following figure;



It is seen that at some point the equipotential surface intersects itself. The intersection point L_1 is one of the Lagrange points of the system (L_2 and L_3 being two other Lagrange points). This implies that a test particle can slide across L_1 from M_1 (or M_2) toward M_2 (or M_1). The shaded regions in the figure, representing two lobes, are called the Roche's lobes of the two masses.

Accretion of mass in a binary system can now be understood in the context of Roche's lobes. If one of the objects, say M_2, is big enough to fill its Roche's lobe, then the mass can accrete from its outer layers to the other object (M_1). The average radius of the Roche's lobe of M_2 is given by:

$$\frac{R_2}{a} \fallingdotseq 0.38 + 0.20 \log q \qquad 0.5 \leq q < 20$$

$$\frac{R_2}{a} \fallingdotseq 0.462 \left(\frac{q}{1+q}\right)^{\frac{1}{3}} \qquad 0 < q < 0.5 \qquad \left(q \equiv \frac{M_2}{M_1}\right)$$

To find the average radius of the Roche's lobe of M_1 we need to change $q \to \frac{1}{q}$, thus:

$$\frac{R_1}{a} \simeq 0.38 - 0.20 \log q \qquad 0.05 < q < 2$$

$$\frac{R_1}{a} \simeq 0.462 \left(\frac{1}{1+q}\right)^{\frac{1}{3}} \qquad 2 \leq q$$

Also, the distance of Lagrange point L_1 from the center of M_1 is given by:

$$\frac{b_1}{a} = 0.500 - 0.277 \log q$$

Once the mass accretes from M_2 to M_1, then q will change. As a result, R_1 and R_2 as well as b_1 and a will also change. Therefore the question is whether the accretion will be sustained. Equivalently, whether M_2 will still be bigger than the average radius of its Roche's lobe. To examine the situation, we assume the following:

(1) $M = M_1 + M_2 = \text{Const.}$

(2) $\vec{L} = \vec{L_1} + \vec{L_2} = \text{Const.}$

We ignore processes like stellar winds that can transfer mass and angular momentum away from the binary system.

From the conservation of the total mass we have:

$$M_1 (1+q) = \text{Const.}$$

Conservation of angular momentum results in:

$$M_1 a_1^2 \omega + M_2 a_2^2 \omega = \text{Const.}$$

Here a_1 and a_2 are the distances of M_1 and M_2 from the center-of-mass respectively, which are given by:

$$a_1 = \frac{qa}{1+q} \quad , \quad a_2 = \frac{a}{1+q}$$

Since $\omega = \frac{2\pi}{T_{orb}}$, we find:

$$\frac{q}{1+q} \frac{M_1 a^2}{T_{orb}} = \text{Const.}$$

Using the Kepler's law, we have:

$$a^2 \propto T_{orb}^{4/3} \Rightarrow T_{orb} \propto \frac{(1+q)^6}{q^3} \propto \frac{1}{M_1^3 M_2^3}$$

Thus,

$$a \propto \frac{(1+q)^4}{q^2} \propto \frac{1}{M_1^2 M_2^2}$$

Note that both a and T_{orb} are minimum when $q=1$.

If $q>1$, reduction of q because of the accretion also results in decrease of a (the opposite occurs when $q<1$). Using the equation for R_2, we then see that:

$$R_2 = f(q)\, a \qquad f(q) = \begin{cases} 0.38 + 0.20 \log q & 0.5 \le q < 20 \\ 0.462 \left(\frac{q}{1+q}\right)^{1/3} & 0 < q < 0.5 \end{cases}$$

$$\frac{\Delta R_2}{R_2} = \left(\frac{f'}{f} + \frac{a'}{a}\right) \Delta q$$

It can be seen that $\left|\frac{a'}{a}\right| > \left|\frac{f'}{f}\right|$, and hence:

$$\frac{\Delta R_2}{R_2} \sim \frac{a'}{a} \Delta q = \frac{\Delta a}{a}$$

We see that $\Delta a < 0$ implies that $\Delta R_2 < 0$, while $\Delta a > 0$ leads to $\Delta R_2 > 0$. Having $\Delta R_2 < 0$ guarantees that accretion will be self-sustained. This requires $\Delta a < 0$ due to accretion, which happens when $q > 1$.

We conclude that accretion will be sustained as long as $M_2 > M_1$. I.E., a compact object accreting from its big companion. In real systems, other effects can sustain accretion even if $q < 1$. For example, if the companion star is out of equilibrium, as would occur after mass loss, its outer envelope swells and fills its Roche's lobe, even if its average radius decreases. This can further drive the binary toward a more compact object.